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Validity of the modified fatigue strength ratio and SWT Parameter for Woven – Roving GFRP under in-phase and out-of-phase combined loading

M.Elhadary

Abstract—the modified fatigue strength ratio (Ψ) and the Smith-Watson-Topper (SWT) parameter have allowed a great reduction in the number of tests required to estimate the effect of mean stress on the fatigue life of some composites for which they were validated. The experimental fatigue tests were conducted on thin-walled tubular specimens woven-roving glass fiber reinforced polyester (GFRP), with 0o, 45o and 90o phase shift between bending and torsional moments for two fiber orientations, ($[\pm45]_2$ and $[0,90]_2$), at different negative stress ratios, R = - 1, -0.75, -0.5, -0.25, 0. It was found that both parameters are valid for studied case, resulting in a great reduction in time and effort when studying a similar case in future.

Index Terms— Fatigue, Glass fiber, Polyester, Combined moments, Fluctuating stresses, SWT Parameter, Modified fatigue strength ratio, Out-of-phase.

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1. INTRODUCTION

One of the important targets of the fatigue work is to find an effective measure for fatigue data analysis to establishing the master S-N relationship that is independent of both stress ratio and fiber orientation in order to predict the fatigue at any arbitrary stress ratio or fiber orientation. The simplest non-dimensional scalar measure for fatigue strength can be defined as $\psi = \frac{\sigma_{max}}{\sigma_b}$ which is usually called the fatigue strength ratio, where σ_{max} is the maximum fatigue strength, which is a function of number of cycles to failure, and σ_b is the static strength. The fatigue strength ratio ψ becomes an effective measure for fatigue data analysis, which was confirmed by Basquin [1] for the fatigue behavior of metals and by Awerbuch and Hahn [2] for the off-axis fatigue behavior of unidirectional graphite/ epoxy composites at room temperature. Since the fatigue strength ratio ψ is defined using only the maximum fatigue stress, it is insensitive to any difference in the waveform of fatigue loading. In order to incorporate the sensitivity to different modes of loading, the stress ratio R, defined as $R = \frac{\sigma_{\min}}{\sigma_{\max}}$ was used by Kawai [3], He decomposed $\sigma_{\max}$ into two parts as

$$\sigma_{\max} = \sigma_a + \sigma_m$$

Where $\sigma_a and \sigma_m$ represent the alternating stress and mean stress, respectively, and are expressed as:

$$\sigma_a = \frac{1}{2}(1-R)\sigma_{\max} \qquad \& \qquad \sigma_m = \frac{1}{2}(1+R)\sigma_{\max}$$

Using $\sigma_a and \sigma_m$, he confirmed that the static failure condition $\sigma_{\max} = \sigma_b$ is expressed as $\frac{\sigma_a}{(\sigma_b - \sigma_m)} = 1$. And by analogy to ψ, therefore he defined a non-dimensional scalar quantity Ψ as $\Psi = \frac{\sigma_a}{\sigma_b - \sigma_m}$, or with the help of R, Ψ can be expressed as

$$\Psi = \frac{\frac{1}{2}(1-R)\psi}{1-\frac{1}{2}(1+R)\psi}$$

The non-dimensional scalar measure Ψ was called the modified fatigue strength ratio. As for fatigue behavior of metals, it is demonstrated by Landgraf [4] that the mean stress effect can be accounted for using the modified fatigue strength ratio Ψ in the fatigue data analysis. Kawai [3] studied the validity of Ψ for the off-axis fatigue behavior of unidirectional composites, using the experimental results of El Kadi and Ellyin [5] conducted on a unidirectional glass/epoxy composites with 0°, 19°, 45°, 71°, 90° fiber orientations at variety of stress ratios, R = -1.0, 0, 0.5 as plots of the maximum fatigue stress $\sigma_{\max}$ and modified fatigue strength ratio Ψ against the number of reversals to failure on logarithmic scales, he observed that:

- The off-axis S-N relationship plotted using the maximum fatigue stress $\sigma_{\max}$ depends on both fiber orientations and stress ratios.

- *Mechanical Eng. Dept., Faculty of Eng., Alexandria University, Alexandria, Egypt*

- The modified fatigue strength ratio Ψ eliminated the fiber orientation dependence as well as the mean stress dependence.

Finally he confirmed that the modified fatigue strength ratio Ψ becomes a unified strength measure to cope with the mean stress effect as well as the fiber orientation effect on the off-axis fatigue behavior of unidirectional composites undergoing constant amplitude stress cycling over a range of stress ratios. The SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ proposed by Smith et al [6] has allowed a reduction in the number of tests needed to estimate the effects of mean stress on the fatigue life of metals. They found that, when plotting the fatigue life against the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$, the data from a test series with variable mean stress or R ratios $\left(\frac{\sigma_{\min}}{\sigma_{\max}}\right)$ tended to plot a single curve. It was subsequently possible to characterize the mean stress effect on fatigue life for a given metal with only a single fatigue test (usually fully reversed, R = -1). Mohamed N.A. [7] tried to check the validity of the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ for composite materials subjected to torsional moments with different R. He tested woven-roving glass fiber-reinforced polyester specimens with [0,90]2 and [±45]2 fiber orientations under torsion fatigue loading with negative stress ratios (R); R=-1, -0.75, -0.5, 0, in plotting the SWT parameter for each fiber orientations against the number of cycles to failure. Also, he used the data for the work of Sharara A. I. [8] since the specimens had nearly the same specifications as those of his work, being woven-roving GFRP tubular specimens with volume fraction (V_f) ranging from 50% to 64% and two fiber orientations $[0,90]_2$ and $[\pm45]_2$ tested under uniaxial bending stress with the same stress ratios. He found that the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ is valid for woven-roving glass fiber-reinforced polyester specimens with $[0,90]_2$ and $[\pm45]_2$ fiber orientations under negative stress ratios for both works, i.e. performing only the completely reversed fatigue test (R=-1) and using the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ is sufficient to find out the strength of the materials under any negative stress ratio.

Sauer J. A. et al. [9] examined un-reinforced axially loaded polystyrene samples, at several tensile mean stress values. They made two groups of tests; the first one was with constant amplitude stress and different mean stresses resulting in varying maximum stresses, and the second group was with constant maximum stress with different combinations of mean and amplitude stresses. When plotting the test results, they used three forms of S-N curves; they used ($\sigma_{\max}$), (σ_a) and ($\sqrt{\sigma_{\max}\sigma_a}$) as the ordinate versus the number of cycles to failure as abscissa. The plots indicated that, using the ($\sqrt{\sigma_{\max}\sigma_a}$), which is the SWT parameter, is slightly better than using $\sigma_{\max}$ and both are better than σ_a. This was because ($\sqrt{\sigma_{\max}\sigma_a}$) succeeded in showing the effect of mean stress in both types of tests. Ryder J. T. and Walker E. K. [10] performed axially loaded fatigue tests with various tensile mean stresses on un-notched quasi-isotropic T300 / 934 graphite / epoxy specimens. Using the same procedure as Sauer J. A. et al. [9], they found that, in this case, both (σ_a) and ($\sqrt{\sigma_{\max}\sigma_a}$) represent the data equally well and better than $\sigma_{\max}$. Finally, both Sauer et al. [9] and Ryder and Walker [10] have found that the SWT parameter can be used exactly in the same manner as it is applied in metals, when expecting a tensile failure mode.

Schuetz D. and Gerharz E. J. [11] studied the possibility of differences between tensile and compressive failure mechanisms in axially loaded carbon fibre reinforced plastic. Their tests covered the range of stress ratios from 0.1 to –5, and indicated that the compressive mean stress results are not well represented by the ($\sqrt{\sigma_{\max}\sigma_a}$) parameter. When they used ($-\sqrt{|\sigma_{\min}|\sigma_a}$) instead, the results were better represented, but the lack of further information regarding the failure modes made it impractical to ensure the use of this new form. Other investigations were done successfully on using the form ($\sqrt{\sigma_{\max}\sigma_a}$) in tension and ($-\sqrt{|\sigma_{\min}|\sigma_a}$) in compression. Ramani S. V. and Williams D. P. [12] checked their applicability on [0,30o] graphite / epoxy specimens and Sturgeon J. B. and Rhodes F. S. [13] on [±45°] carbon / epoxy specimens. They both found that using ($\sqrt{\sigma_{\max}\sigma_a}$) for tensile and zero mean stresses and ($-\sqrt{|\sigma_{\min}|\sigma_a}$) for compressive mean stresses work very well for the studied cases.

2. EXPERIMENTAL WORK

Woven-roving E-glass fibers and polyester resin, with trade name of “siropol 8330”, were used to produce the used specimens. Table 1 shows the properties of the tested materials [ASTM. D2150-63]. This resin was prepromoted with Cobalt Naphthenate (6% solution), as an accelerator in percentage of 0.2 % by volume, and Methyl Ethyl Ketone (M.E.K.) peroxide as a catalyst in a percentage of 2% by volume, depending on room temperature. The volume fraction (Vf), in the present work, ranges from 55% to 65% was used; because this range has proved its suitability to ensure specimens with good strength, good adhesion between fibers and matrix, and acceptable mechanical properties.

TABLE 1
PROPERTIES OF USED MATERIALS

Woven–roving E-glass fibers		Polyester	
Property	Value	Property	Value
Density	2551 kg /m3	Density	1161.3 kg / m3
Modulus of elasticity	E = 76 GPa	Modulus of elasticity	E = 3.5 GPa
Poisson's ratio	υ = 0.37	Poisson's ratio	υ = 0.25
Tensile strength	3.45 GPa	Gel time at 25o C	20 min.
Average mass / area	600 gm / m2	Viscosity	0.45 Pa.s
Average thickness	0.69 mm	Percentage of Styrene	40 %
Weave	plain	Trade name	Siropol 8340

Thin-walled tubular specimens were used for the experimental work to ensure having a plane uniform stress. Fig.1 shows the dimensions of the used specimens. These dimensions are similar to those used by pervious investigators [6,7]. To avoid the failure of some specimens at the end of the gauge length, beneath the grippers, two wooden plugs were inserted into the specimens from both ends. And an elastic sleeve was shrinked on the outer diameter at both ends. A strain controlled testing machine, previously designed by Abouelwafa M.N. et al [8]. It is a constant speed machine of 525 rpm (8.75 Hz); and capable of performing pure torsion, pure bending, or combined torsion and bending (in-phase or out-of-phase) fatigue tests. Fig. 2. shows a general layout for the machine, the loading systems (torsion and bending) are independent, and have the facility to apply different mean stresses.

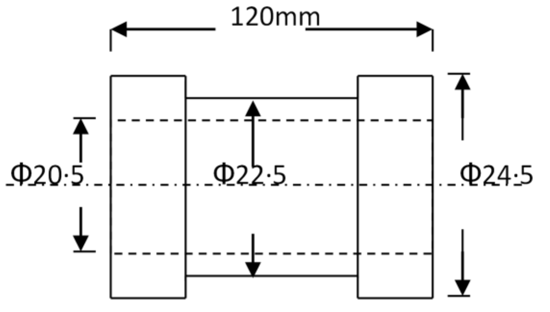

Fig.1 Dimensions of used specimens

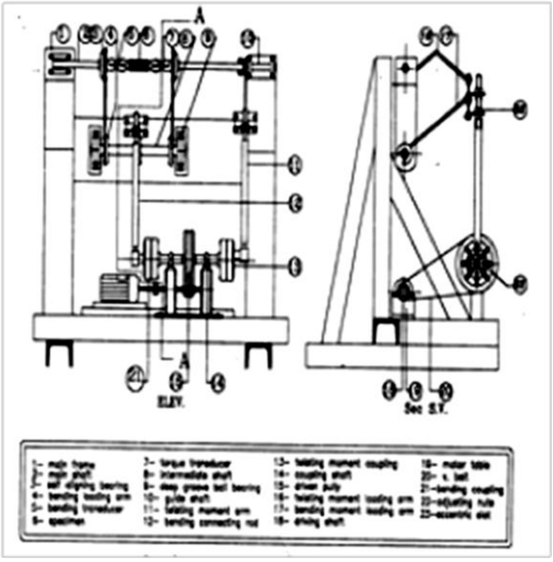

Fig.2 General layout of the testing machine

3. TEST RESULTS

Specimens were fatigue tested under ambient conditions and constant frequency of 8.75 Hz. For each orientation $[0,90]_2$ and $[\pm45]_2$, the data points were used to plot the corresponding S-N curves on a semi-log scale, being fitted using the power low: Max. Stress = a N^b
Table2 lists the two constants a and b for the corresponding S-N curve for $[0,90]_2$, tested in-phase, with different stress ratios (R = -1, -0.75, -0.5, -0.25, 0) at the ratio of the flexural stress (A) to the torsional shear stress (B), A/B= 2, while Table3 is for $[\pm45]_2$ at A/B=1. In order to study the effect of mean stress in the presence of a phase difference between the bending and the torsional moments (Z), tests were performed on both fiber orientations, $[0,90]_2$ and $[\pm45]_2$, at four different stress ratios (R = -1, -0.75, -0.5, -0.25). Table 4 lists the two constants a and b for the corresponding S-N curves for $[0,90]_2$ at all stress ratios with A/B=C/D=2 and Z=45^o and 90^o, while Table5 is for $[\pm45]_2$ at A/B=1.

TABLE 2
FATIGUE CONSTANTS (A) AND (B) OF $[0,90]_2$ SPECIMENS WITH A/B=2 & Z=0^O

Stress ratio (R)	a. (MPa)	b.	Correlation factor
-1	103.93	-0.1286	0.9748
-0.75	113.92	-0.1271	0.9967
-0.5	126.11	-0.1266	0.994
-0.25	141.85	-0.1267	0.9965
0	170.17	-0.1265	0.9837

TABLE 3

FATIGUE CONSTANTS (A) AND (B) OF $[\pm45]_2$ SPECIMENS WITH A/B=1 & Z=0°

Stress ratio (R)	a. (MPa)	b.	Correlation factor
-1	103.93	-0.1286	0.9748
-0.75	113.92	-0.1271	0.9967
-0.5	126.11	-0.1266	0.994
-0.25	141.85	-0.1267	0.9965
0	170.17	-0.1265	0.9837

TABLE 4

FATIGUE CONSTANTS (A) AND (B) OF $[0,90]_2$ SPECIMENS WITH A/B=2

R	Z=90°		Z=45°	
	a.	b.	a. (MPa)	b.
-1	89.51	-0.112	95.83	-0.119
-0.75	93.47	-0.106	100.41	-0.114
-0.5	104.04	-0.108	113.18	-0.115
-0.25	109.52	-0.103	121.04	-0.113

4. VALIDITY OF MODIFIED FATIGUE STRENGTH RATIO (Ψ)

The fatigue tests with different stress ratios; for the $[0,90]_2$ specimens with (A/B=2) and for the $[\pm45]_2$ specimens with (A/B=1) were used to plot the modified fatigue strength ratio (Ψ) for each of the two fiber orientations against the number of cycles to failure, using the power-law form:
$\Psi = a_2 N^{b2}$ for fitting these data points, as shown in Fig.3 to Fig.7. Table 5. gives the values of the curve fitting constants a2 and b2 with acceptable correlation factors.

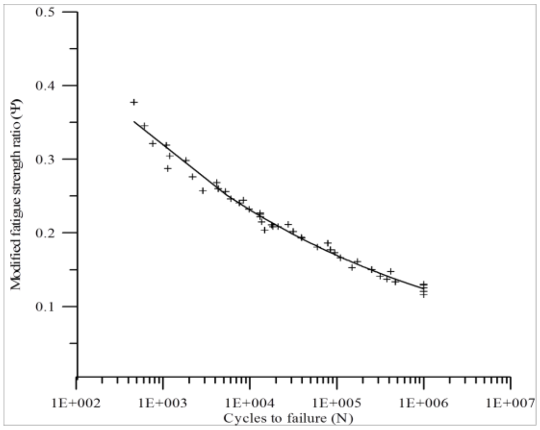

Fig.3 R.D Modified fatigue strength ratio Ψ for $[0,90]_2$ specimens at A/B=2 & Z=0°

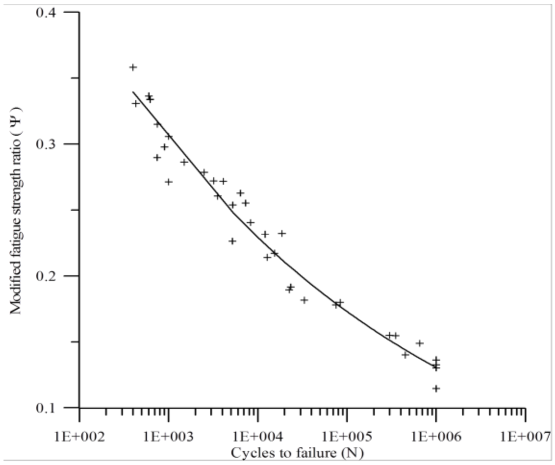

Fig.4 R.D Modified fatigue strength ratio Ψ for $[0,90]_2$ specimens at A/B=2 & Z=45°

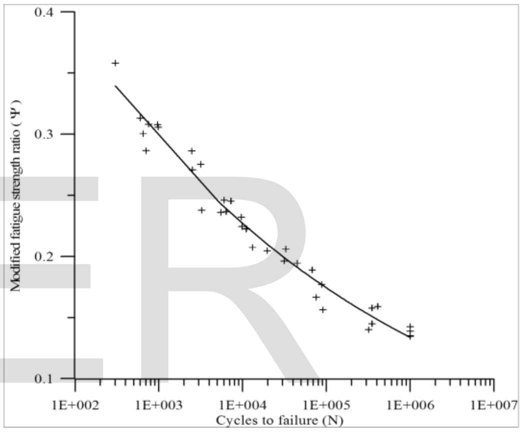

Fig.5 R.D Modified fatigue strength ratio Ψ for $[0,90]_2$ specimens at A/B=2 & Z=90°

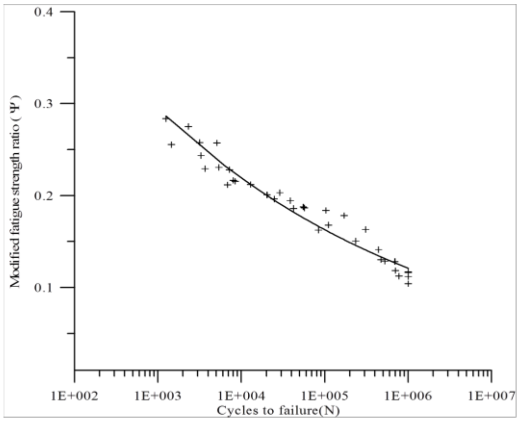

Fig.6 R.D Modified fatigue strength ratio Ψ for $[\pm45]_2$ specimens at A/B=1 & Z=0°

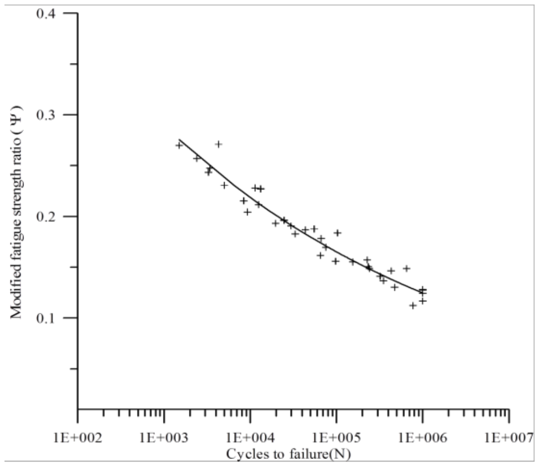

Fig.7 R.D Modified fatigue strength ratio Ψ for $[\pm45]_2$ specimens at A/B=1 & Z=90°

TABLE 5
FATIGUE VALUES OF CONSTANTS (a_2) AND (b_2) FOR $\Psi = a_2 N^{b_2}$

	Type	$\Psi = a_2 N^{b2}$		Correlation Factor	$\sigma_{max.}=aN^b$ R=-1	$\left(\frac{a}{S_u}\right)$
		a_2	b_2			
$[0,90]_2$	Z=0°	0.74	-0.135	0.987	$104\ N^{-0.128}$	0.73
	Z=45°	0.706	-0.122	0.972	$95\ N^{-0.112}$	0.69
	Z=90°	0.65	-0.114	0.971	$89\ N^{-0.119}$	0.64
$[\pm45]_2$	Z=0°	0.718	-0.129	0.943	$84.6\ N^{-0.129}$	0.7
	Z=45°	0.69	-0.125	0.981	$82.69\ N^{-0.127}$	0.701
	Z=90°	0.67	-0.12	0.949	$77.94\ N^{-0.112}$	0.65

From Table V. we can notice that:

- The deviation in the values of (b_2), at different loading conditions for both fiber orientations $[0,90]_2$ and $[\pm45]_2$, is small. So it may be considered to be material constant: $b_2 = -0.124$ with standard deviation =0.73 %
- The value of (a_2) was found to be nearly equal to the ratio (a/ Su).

Where:

Su: the static strength in fiber direction

$\{(S_u)_{[\pm45]} = 118$ MPa, $(S_u)_{[0,90]} = 137$ MPa$\}$

a: the constant of (σmax. = a N^b) at completely reversed load (R=-1) for each specified loading condition.

5. VALIDITY OF SWT PARAMETER

The tested specimens with different stress ratios were used to plot the SWT parameter for each of the two fiber orientations, against the number of cycles to failure. The values of SWT parameter at different stress ratios (R) were obtained from the following equation:

$$SWT = \sigma_{\max}\sqrt{\frac{1-R}{2}}$$

Fig.8 to Fig.12 show these results for the $[0,90]_2$ and the $[\pm45]_2$ specimens. Using the power-law form: $SWT = a_1 N^{b1}$ for fitting these data points, the values of the constants, (a_1) and (b_1), are given in Table 6.

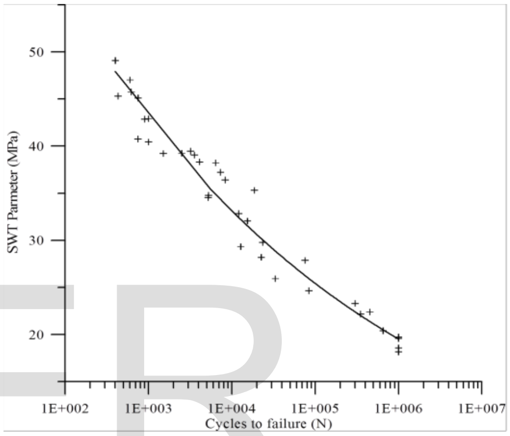

Fig.8 SWT parameter for $[0,90]_2$ specimens at A/B =2 & Z=0°

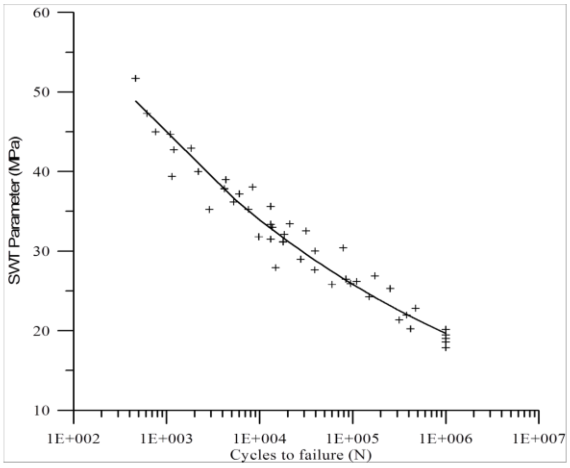

Fig.9 SWT parameter for $[0,90]_2$ specimens at A/B =2 & Z=45°

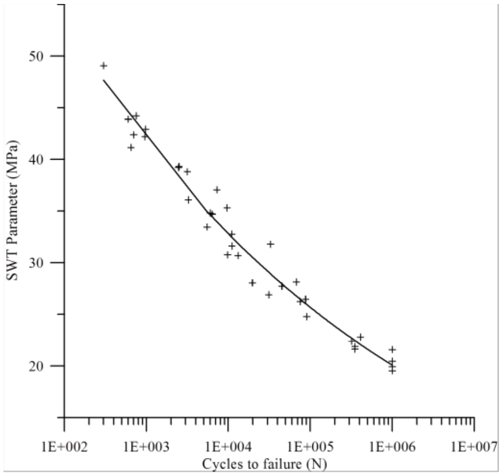

Fig.10 SWT parameter for $[0,90]_2$ specimens at A/B =2 & Z=90°

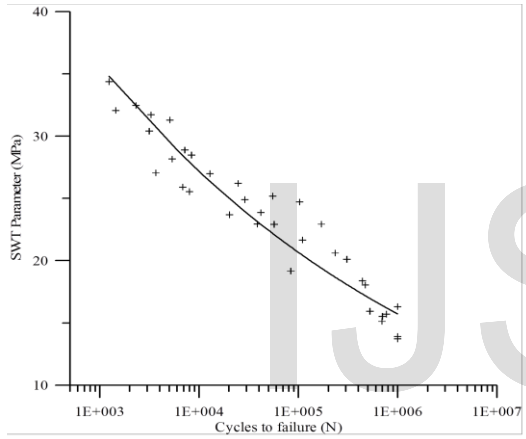

Fig.11 SWT parameter for $[\pm45]_2$ specimens at A/B =1 & Z=0°

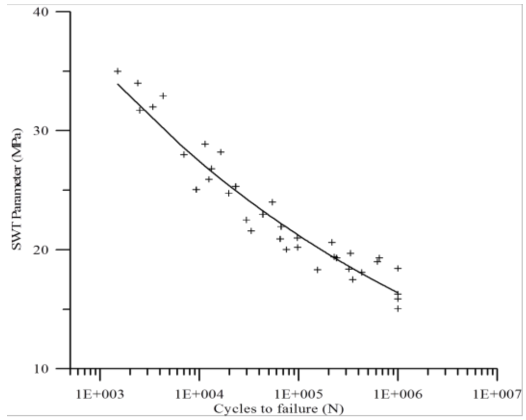

Fig.12 SWT parameter for $[\pm45]_2$ specimens at A/B =1 & Z=90°

TABLE 6
VALUES OF CONSTANTS (a1) AND (b1) FOR SWT PARAMETER

Type		SWT=a_1N^{b1}	Correlation Factor	$\sigma_{max.}$=aN^b R=-1	$\left(\frac{a_1}{S_u}\right)$
$[0,90]_2$	Z=0°	101.1 $N^{-0.12}$	0.951	104 $N^{-0.128}$	0.73
	Z=45°	95.3 $N^{-0.115}$	0.965	95 $N^{-0.112}$	0.69
	Z=90°	87.6 $N^{-0.11}$	0.971	89 $N^{-0.119}$	0.64
$[\pm45]_2$	Z=0°	83.17 $N^{-0.12}$	0.907	84.6 $N^{-0.129}$	0.7
	Z=45°	82.83 $N^{-0.124}$	0.968	82.69 $N^{-0.127}$	0.701
	Z=90°	76.84 $N^{-0.112}$	0.924	77.94 $N^{-0.112}$	0.65
	Z=90°	76.84 $N^{-0.112}$	0.924	77.94 $N^{-0.112}$	0.65

Table 6 shows that: The ratio between the constant (a_1) to the static strength of the corresponding fiber orientation is nearly equal for both orientations at different phase angels, this conclusion was also found by Mohamed N.A. [7]. Comparing the values of the two constants (a_1) and (b_1) to the corresponding two constants (a) and (b) of the corresponding fiber orientation resulted in a good promising result.

This means that the SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ can be used for woven-roving GFRP with $[0,90]_2$ and $[\pm45]_2$ orientations under combined bending and torsion in-phase and out-of-phase fatigue loading with negative stress ratios. Performing only the completely reversed (R=-1) fatigue test and using the SWT parameter will be sufficient to find out the strength of the material under any negative stress ratio.

6. CONCLUSIONS:

- The modified fatigue strength ratio (Ψ) has become a useful measure for establishing the master S-N relationship for woven-roving GFRP with $[0,90]_2$ and $[\pm45]_2$ orientations under combined bending and torsion in-phase and out-of-phase fatigue loading with negative stress ratios. Using the power formula $\Psi = a_2 N^{b2}$ has proved its suitability for $[0,90]_2$ and $[\pm45]_2$ specimens. And the value of (b_2), at different loading conditions for both fiber orientations $[0,90]_2$ and $[\pm45]_2$, may be considered to be material constant. The value of (a_2) was found to be nearly equal to the ratio (a/ su), where Su is the static strength in fiber direction and (a) is the constant of (σmax. = a N^b) at completely reversed load (R= -1) for each specified loading condition.

- The SWT parameter $\left(\sqrt{\sigma_{\max}\sigma_a}\right)$ is valid for woven-roving GFRP with $[0,90]_2$ and $[\pm45]_2$ orientations under combined bending and torsion in-phase and out-of-phase fatigue loading with negative stress ratios. Performing only the completely reversed (R=-1) fatigue test and using the SWT parameter will be sufficient to find out the strength of the material under any negative stress ratio. Using the power formula SWT = a_1 N^{b1} has resulted in having a nearly constant ratio between (a_1) and the corresponding static strength for both fiber orientations.

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